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# Kalman Filter in the Strapdown Airborne Gravimetry Problem Based on the Refined Model of GNSS Data Errors

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**Abstract**—The paper considers the problem of gravity disturbance determination on the aircraft flight trajectory using measurements from a strapdown airborne gravimeter. The gravimeter measurements include raw data from the inertial sensors and global navigation satellite system (GNSS) receivers. The problem is reduced to optimal stochastic estimation given an a priori model of gravity disturbance in the time domain and stochastic models of the inertial sensor measurement errors and the GNSS data errors (the errors of kinematic accelerations derived from carrier phase measurements). The estimation algorithm is the Kalman filter and smoothing. We show that the accuracy of gravity estimation can be improved when using a refined model of the kinematic acceleration errors instead of using the traditional model (a white noise process). The refined model is given as the second difference of a discrete-time white noise.

*Keywords*: airborne gravimetry, strapdown gravimeter, GNSS, kinematic acceleration errors, optimal estimation, Kalman filter

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## 1. INTRODUCTION

Strapdown airborne gravimetry aims to determine gravity disturbance from measurements of a strapdown gravimeter on the flight trajectory of an aircraft (fixed-wing aircraft, helicopter or drone). The gravity disturbance is the difference between the magnitudes of the real gravity vector and normal gravity vector. The normal gravity is defined for the ellipsoid model of the Earth [1].

A strapdown airborne gravimeter consists of a strapdown inertial navigation system or inertial measurement unit (IMU), which includes high-precision inertial sensors (accelerometers and gyroscopes) and geodetic-grade global navigation satellite system (GNSS) receivers (onboard and ground-based). Raw gravimeter data includes measurements from the IMU inertial sensors and GNSS receivers and are processed following the steps of a postprocessing scheme [2] (see also, e.g., [3–5]):

(1) computing GNSS solutions (determining position, velocity, and accelerations of the aircraft from raw GNSS measurements);
(2) computing integrated IMU/GNSS solutions (estimating the attitude of the gravimeter's IMU, the instrumental errors of the IMU inertial sensors, GNSS antenna offsets, etc.);
(3) computing gravimetric solution (determining the gravity disturbance along the flight trajectory).

In the first stage of the postprocessing scheme, GNSS solutions are calculated using code pseudorange, Doppler pseudorange rate and carrier-phase measurements from dual-frequency receivers accessing signals from one or several satellite constellations (GPS, GLONASS, GALILEO, BeiDou, etc.) [4, 5]. The carrier-phase differential mode of data processing is typically used to reduce GNSS measurement errors caused by ionospheric and tropospheric effects. Alternatively, the Precise Point Positioning (PPP) technology, which does not require ground-based receivers (base stations), may be used to obtain high-accuracy GNSS solutions [6]. The purpose of this postprocessing stage is to calculate the aircraft's velocity and/or accelerations with high accuracy, primarily using carrier phase measurements and, less commonly, Doppler pseudorange rate measurements (see, e.g., [5, 7]). Another approach is to use these two types of measurements simultaneously, with Doppler pseudorange rate measurements serving as auxiliary data in case of carrier phase measurement failures [4].

In the second stage of postprocessing (IMU/GNSS integration), the position, velocity and attitude of the gravimeter's IMU computed from the inertial sensor measurements are refined using the GNSS solutions. The mathematical foundation of this task is the IMU error dynamics equations expressed in the axes of the navigation (geodetic) frame (see, e.g., Section 5.2 in [1]). The vertical channel is excluded from the equations, which removes gravity disturbance (as an unknown unknwn variable) from the equations. The influence of gravity disturbance on the horizontal channels, expressed as the product of gravity disturbance and the horizontal attitude errors, is commonly neglected as a second-order small quantity. Next, an optimal stochastic estimation problem is formulated and solved using Kalman filtering and smoothing, which results in the state vector estimates obtained at each point of the aircraft's flight trajectory. In particular, the estimates of the IMU attitude errors and the inertial sensor errors are obtained at this stage (for details, please see [2]).

In this work, we focus on the third stage of postprocessing, which is solving the gravimetry problem (determining gravity on the flight trajectory). The basic idea of solving the problem is to form the difference between the IMU vertical channel data and GNSS data (kinematic acceleration in the vertical direction). The problem is formulated as an optimal stochastic estimation problem given a priori stochastic models of gravity disturbance, sensor measurement errors and GNSS data errors. The solution to the estimation problem is provided by the Kalman filter and smoothing. Traditionally, the GNSS acceleration errors are modeled as white noise [8, 9]. However, these errors typically have a more complex structure and are correlated in time.

In this work, a refined stochastic model of the GNSS acceleration errors is introduced (for the first time to our knowledge) under the assumption that GNSS accelerations are computed using carrier phase measurements (Doppler pseudorange rates are not considered in the paper). Namely, we assume that the kinematic accelerations are computed based on double differences of GNSS carrier phase measurements (involving three successive time epochs) [10]. The refined model of GNSS acceleration errors is defined in the time domain as the second central difference of a discrete-time white noise process. The airborne gravimetry problem is then reduced to a linear optimal stochastic estimation problem and solved in the Kalman filter framework. The results of testing the proposed approach using real airborne gravimeter data are presented and discussed. The results demonstrate a higher accuracy of gravity determination comparing to the traditional approach based on the simplified GNSS acceleration error model (white noise).

## 2. MATHEMATICAL MODELS

### *2.1. Basic Equations*

The following notation is used in this study:

- $M$ is the proof mass of the accelerometer triad of the gravimeter's IMU;

- $Mx$ is the geodetic frame with the origin at the point $M$ and the axes pointing east, north, and up along the normal to the reference ellipsoid (denoted as $E, N, Up$) [1];
- $Mz$ is the IMU body frame with the axes $z_1, z_2, z_3$ mutually perpendicular and aligned with the sensitivity axes of the calibrated IMU accelerometers.

The mathematical foundation of the strapdown airborne gravimetry problem is the equation of motion of the point $M$ expressed in the geodetic frame $Mx$ (for the expressions in other reference frames see, e.g., [1]):

$$\mathbf{a}_x = -(\boldsymbol{\Omega}_x + 2\mathbf{u}_x) \times \mathbf{v}_x + \mathbf{g}_x^0 + \delta\mathbf{g}_x + L_{zx}^{\mathrm{T}}\mathbf{f}_z, \tag{1}$$

where $\mathbf{v}_x, \mathbf{a}_x$ are the velocity and acceleration vectors (relative to the Earth) of the point $M$, respectively, expressed in the geodetic frame $Mx$; $\boldsymbol{\Omega}_x$, $\mathbf{u}_x$ are the angular velocity of $Mx$ relative to the Earth and the angular velocity of the Earth relative to the inertial space, respectively; $\mathbf{g}_x^0 = (0, 0, -g_0)^{\mathrm{T}}$ is the normal gravity vector at the point $M$ [11]; $\delta\mathbf{g}_x$ is the gravity disturbance vector [1]; $\mathbf{f}_z$ is the specific force at the point $M$ expressed in the IMU body frame $Mz$; $L_{zx}$ is the transformation matrix from the geodetic frame $Mx$ to the body frame $Mz$ (an orthogonal matrix).

The vertical projection of (1) is given by the formula (the fundamental equation of airborne scalar gravimetry):

$$a_{up} = g_{etv} - g_0 - \delta g + L_3^{\mathrm{T}}\mathbf{f}_z, \tag{2}$$

where $a_{up}$ is the relative vertical acceleration of the point $M$, $g_{etv}$ is the Eötvös correction term (the vertical projection of the inertial forces), $g_0$ is the magnitude of the normal gravity vector at the point $M$, $\delta g$ is the magnitude of $\delta\mathbf{g}_x$ (gravity disturbance), $L_3$ is the third column of the transformation matrix $L_{zx}$.

The gravimetric problem, as noted earlier, is solved at the final stage of the postprocessing strategy and involves determining the gravity disturbance $\delta g$ on the flight trajectory based on (2) using raw accelerometer measurements, GNSS solutions (positions, velocities and accelerations), and integrated IMU/GNSS solutions (estimates of the IMU attitude).

### *2.2. Measurement Models*

In (2), only the specific force $\mathbf{f}_z$ is measured directly (by the IMU accelerometers). The measurement model is

$$\mathbf{f}_z' = \mathbf{f}_z + \mathbf{q}_f, \tag{3}$$

where $\mathbf{f}_z'$ is the vector of three accelerometer measurements and $\mathbf{q}_f$ is the vector of measurement errors.

The vertical kinematic acceleration of the aircraft is derived from raw (carrier phase) GNSS measurements and can be expressed as

$$a_{up}^{gps} = a_{up} + e_a, \tag{4}$$

where $a_{up}^{gps}$ is the vertical acceleration computed from raw GNSS data and $e_a$ is the acceleration error.

The Eötvös correction term $g_{etv}$ is determined using the position and the horizontal components of the velocity vector at the point $M$ [1]. The normal gravity $g_0$ at the point $M$ is determined using a theoretical model for the normal gravity at the ellipsoid (e.g., Helmert's formula or Somigliana's formula) and the height correction term [11]. For computing the Eötvös correction term and normal gravity, the GNSS position and velocity solutions (or the IMU/GNSS solutions) are used [4]. The lever-arm effect caused by the distance between the onboard GNSS antenna and the IMU should be taken into account.

The estimate $\tilde{L}_3$ of the third column $L_3$ is also assumed to be known and computed from the estimates of the IMU attitude angles (heading, roll and pitch) [12].

The computed vertical acceleration $a'_{up}$ is defined as

$$a'_{up} = g_{etv} - g_0 + \tilde{L}_3^{\mathrm{T}} \mathbf{f}'_z. \tag{5}$$

### *2.3. Fundamental Equation of Airborne Strapdown Gravimetry*

Define the difference between the computed vertical acceleration and the true value as $\Delta a_{up} = a_{up} - a'_{up}$, which can be expressed using (2) and (5) in the following form:

$$\Delta a_{up} = -\delta g + L_3^{\mathrm{T}} \mathbf{f}_z - \tilde{L}_3^{\mathrm{T}} \mathbf{f}'_z. \tag{6}$$

The transformation matrix $\tilde{L}_{zx}$ is computed from the integrated IMU/GNSS solution, that is, from the estimated (refined) IMU attitude angles (heading, roll, pitch), and is an orthogonal matrix. The relationship between the true transformation matrix $L_{zx}$ and the computed matrix $\tilde{L}_{zx}$ can be written as

$$L_{zx} = (I + \hat{\kappa}) \tilde{L}_{zx},$$

where $\hat{\kappa}$ is a skew-symmetric matrix composed of components of the small rotation vector $\kappa = (k_E, k_N, k_{Up})^{\mathrm{T}}$. The vector $\kappa$ characterizes the residual attitude error, that is, the errors in the IMU attitude estimates obtained at the IMU/GNSS integration stage. Here, $k_E, k_N$ are the residual attitude errors in the east and north directions, respectively, and $k_{Up}$ is the error of the azimuthal error estimate.

The right-hand side of (6) can be rewritten using (3) as:

$$L_3^{\mathrm{T}} \mathbf{f}_z - \tilde{L}_3^{\mathrm{T}} \mathbf{f}'_z = (L_3^{\mathrm{T}} - \tilde{L}_3^{\mathrm{T}}) \mathbf{f}'_z - \tilde{L}_3^{\mathrm{T}} \mathbf{q}_f = -k_E f'_N + k_N f'_E - \tilde{L}_3^{\mathrm{T}} \mathbf{q}_f, \tag{7}$$

where $f'_E = \tilde{L}_1^{\mathrm{T}} \mathbf{f}'_z$, $f'_N = \tilde{L}_2^{\mathrm{T}} \mathbf{f}'_z$ are the horizontal projections of the accelerometer measurements in the east and north directions, respectively, and $\tilde{L}_1, \tilde{L}_2$ are the first two columns of the transformation matrix $\tilde{L}_{zx}$.

Using the GNSS-derived vertical acceleration (4), the measurement of $\Delta a_{up}$ can be formed as

$$y := a_{up}^{gps} - a'_{up} = \Delta a_{up} + e_a, \tag{8}$$

where $e_a$ is the error of the GNSS-derived acceleration.

Substituting (7) and (6) into (8), we finally obtain the basic equation of airborne strapdown gravimetry in the form containing measurements and measurement errors:

$$y = -\delta g - k_E f'_N + k_N f'_E - \tilde{L}_3^{\mathrm{T}} \mathbf{q}_f + e_a. \tag{9}$$

Other systematic errors, such as GNSS antenna offsets or time-synchronization errors between the IMU and GNSS data [2], may also be included in (9). However, for simplicity, these errors are not considered here.

Equation (9) is considered over the flight time interval $[t_0, t_n]$. All measurements in (9) ($y$, $f'_E$, $f'_N$, and $\tilde{L}_3$) are assumed to be resampled at the GNSS data rate. Let $t_i$ be a time stamp of GNSS data ($i = 0, \ldots, n$) and $\Delta t$ the time step. The remaining variables in (9) ($\delta g$, $k_E$, $k_N$, $\mathbf{q}_f$, and $e_a$) are treated as unknown functions of time, for which a priori models are introduced below.

## 3. FORMULATION OF THE OPTIMAL ESTIMATION PROBLEM

Problem (9) is reduced to optimal stochastic estimation. A priori stochastic models are introduced for gravity disturbance $\delta g$, residual attitude errors $k_E, k_N$ and GNSS vertical acceleration error $e_a$. The measurement errors of three accelerometers $\mathbf{q}_f$ are modeled as a modeled as stochastic processes, namely, as discrete-time white noises with zero mean and variance $\sigma_f^2$ (it is assumed here that all three accelerometers have the same accuracy).

### *3.1. Stochastic Models of Gravity Disturbance and IMU Systematic Errors*

In airborne gravimetry, it is usually assumed that gravity (as a function of flight time) is a slowly-varying function (with the spectrum mostly concentrated at low frequencies) [13]. An a priori stochastic model of gravity as a stationary random process in time is typically introduced. The models commonly used in airborne gravimetry algorithms are Gauss–Markov models (typically of order two or three) [14, 15], integrals of a white noise [3, 4, 13], and Jordan's model [16]. Deterministic spatial models are also occasionally used in airborne gravimetry [17, 18]. A detailed comparison of all these models is beyond the scope of this paper, but a partial comparison can be found in [15] (Section 5.2) and [14, 15, 17, 18]. For instance, [15] notes that using the Gauss–Markov models of different orders yields similar results in airborne gravimetry as when using the integrals of a white noise.

In this work, gravity disturbance is modeled as the second integral of a white-noise process. The model takes into account the long-wavelength nature of gravity, agrees well with real gravimetric data in many areas [13] and is defined by a simple equation in the time domain: $\delta\ddot{g} = q_g$, where $q_g$ is a white noise. The power spectral density (PSD) of the gravity model is:

$$S_g(\omega) = \frac{\sigma^2}{2\pi\omega^4}, \tag{10}$$

where $\omega$ is the angular frequency and $\sigma^2$ is the intensity of white noise.

Let us write the equations of the gravity model in discrete time denoting with the subscript $i$ the value at the time moment $t_i$:

$$\begin{cases} \delta g_{i+1} = \delta g_i + \Delta t\, p_i, \\ p_{i+1} = p_i + q_{g,i}, \end{cases} \tag{11}$$

where $q_{g,i}$ is a discrete-time white noise with zero mean and variance $\sigma_g^2$.

Further, we introduce the stochastic models for the residual attitude errors $k_E$, $k_N$ in east and north directions. Recall that the IMU attitude errors are estimated at the IMU/GNSS integration stage. The horizontal attitude errors contain the so-called Schueler oscillations [12] and in absolute value typically do not exceed 0.5 arcmin when using a state-of-the-art IMU [2]. The residual attitude errors $k_E$, $k_N$ do not contain the Schueler oscillations, are typically less than 10 arcsec in absolute value and can be modeled as slowly varying functions of flight time [2, 4].

Based on the above, we introduce the models of $k_E$, $k_N$ as integrals of white noise: $\dot{k}_E = q_E$, $\dot{k}_N = q_N$, or in discrete time as:

$$\begin{cases} k_{E,i+1} = k_{E,i} + q_{E,i}, \\ k_{N,i+1} = k_{N,i} + q_{N,i}, \end{cases} \tag{12}$$

where $q_{E,i}$, $q_{N,i}$ are discrete-time white noises with zero mean and variances $\sigma_E^2, \sigma_N^2$, respectively.

### *3.2. Refined Model of Kinematic Acceleration Error*

Traditionally, in airborne gravimetry algorithms the error in GNSS-derived accelerations is assumed to be a white noise. In this work, a refined model of the acceleration error is introduced taking into account the specifics of the acceleration computation method. Namely, we assume that the kinematic accelerations are computed based on numerical differentiation of carrier phase measurements (by forming double differences of measurements using three successive epochs $t_{i-1}$, $t_i$, $t_{i+1}$) [10]. Taking this into account, we introduce the refined model of GNSS acceleration error in the following form:

$$e_{a,i} = \frac{q_{a,i+1} - 2q_{a,i} + q_{a,i-1}}{\Delta t^2}, \tag{13}$$

where $q_{a,i}$ is a discrete-time white noise with zero mean and variance $\sigma_a^2$.

The autocorrelation function of the process $e_{a,i}$ denoted by $K_e(m)$ ($m$ is an integer) takes the following values:

$$K_e(0) = \frac{6\sigma_a^2}{\Delta t^4}, \quad K_e(\pm 1) = -\frac{4\sigma_a^2}{\Delta t^4}, \quad K_e(\pm 2) = \frac{\sigma_a^2}{\Delta t^4},$$

and zero for other $m$.

The PSD of the process $e_{a,i}$ is given by (assuming $\Delta t = 1$ for simplicity):

$$S_e(e^{j\omega}) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} K_e(m)\, e^{-j\omega m} = \frac{2\sigma_a^2}{\pi}(1 - \cos\omega)^2, \tag{14}$$

where $j$ is the imaginary unit.

Figure 1 shows the PSD of the refined error model (13)–(14) and PSD of real kinematic accelerations from a static test (the GNSS receiver from JAVAD with the sampling rate of 10 Hz was used). The recording was made while the aircraft was at the parking position at the airport (see

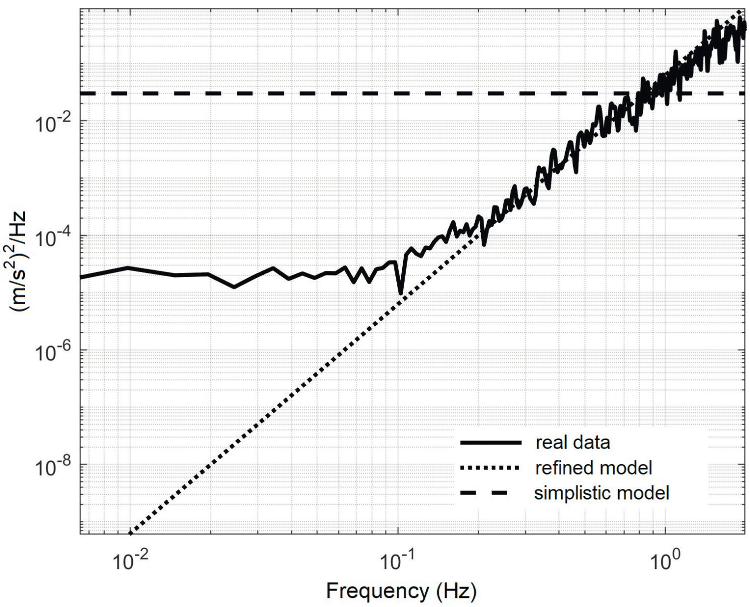


**Fig. 1.** Power spectral densities of GNSS carrier-phase acceleration errors (real data; solid line) and of theoretical models of GNSS acceleration errors: refined model (dotted line) and traditional simplistic model (dashed line), $(\mathrm{m}^2/\mathrm{s}^4)$/Hz.

Section 4 for details). The raw GNSS data processing (acceleration computation) was performed using the software developed by Lomonosov Moscow State University [10]. Since the PSD of the refined model (14) is proportional to $\omega^4$ at low frequencies, its plot is shown in Fig. 1 as a straight line (in a logarithmic scale).

In Fig. 1, the PSD of the simplified GNSS acceleration error model (white noise), which is traditionally used in airborne gravimetry algorithms, is also shown. From Fig. 1, it follows that the refined model matches the real data significantly better, both in the high-frequency domain and near the cutoff frequency of the gravimetric filter [1], that is, in the range of 0.01–0.1 Hz, than the simplified model.

Further, a state-space representation of the refined model (13) will be required. Let us introduce auxiliary variables $\xi_i$, $\eta_i$:

$$\begin{cases} \eta_{i+1} = \xi_i, \\ \xi_{i+1} = q_{\xi,i}, \end{cases} \tag{15}$$

where $q_{\xi,i} := q_{a,i+1}$. Rewriting the model (13) using the auxiliary variables, we obtain:

$$e_{a,i} = \frac{\eta_i - 2\xi_i + q_{\xi,i}}{\Delta t^2}. \tag{16}$$

### *3.3. Problem Statement and Estimation Algorithm*

Let us combine the basic equation of airborne gravimetry (9), the stochastic models of the gravity disturbance (11), the residual attitude errors (12) and the refined model of GNSS acceleration error (15)–(16) into one state-space system:

$$\begin{cases} k_{E,i+1} = k_{E,i} + q_{E,i}, \\ k_{N,i+1} = k_{N,i} + q_{N,i}, \\ \delta g_{i+1} = \delta g_i + \Delta t\, p_i, \\ p_{i+1} = p_i + q_{g,i}, \\ \eta_{i+1} = \xi_i, \\ \xi_{i+1} = q_{\xi,i}, \\ y_i = -\delta g_i - k_{E,i} f'_{N,i} + k_{N,i} f'_{E,i} + \dfrac{1}{\Delta t^2}(\eta_i - 2\xi_i + q_{\xi,i}) - \tilde{L}^{\mathrm{T}}_{3,i}\mathbf{q}_{f,i}. \end{cases} \tag{17}$$

The state-space system can be written in matrix form:

$$\begin{cases} \mathbf{x}_{i+1} = A_i\mathbf{x}_i + B_i\mathbf{q}_i, \\ y_i = C_i\mathbf{x}_i + r_i, \end{cases} \tag{18}$$

where the state vector $\mathbf{x}_i$ has the following form

$$\mathbf{x}_i = (k_{E,i}, k_{N,i}, \delta g_i, p_i, \eta_i, \xi_i)^{\mathrm{T}} \in \mathbb{R}^6. \tag{19}$$

The vector $\mathbf{q}_i$ is the system noise vector:

$$\mathbf{q}_i = (q_{E,i}, q_{N,i}, q_{g,i}, q_{\xi,i})^{\mathrm{T}} \in \mathbb{R}^4. \tag{20}$$

The scalar $r_i$ is the measurement noise:

$$r_i = \frac{1}{\Delta t^2} q_{\xi,i} - \tilde{L}^{\mathrm{T}}_{3,i}\mathbf{q}_{f,i}. \tag{21}$$

The matrices $A_i$, $B_i$, $C_i$ in (18) consist of the coefficients at the unknown variables and noises in (17) and have dimensions $6 \times 6$, $6 \times 4$ and $1 \times 6$, respectively.

The covariance matrix of the system noise vector (20) $E[\mathbf{q}_i \mathbf{q}_i^{\mathrm{T}}]$ is a diagonal $4 \times 4$-matrix whose diagonal elements are the variances of the components of $\mathbf{q}_i$. The variance of the measurement noise $r_i$ (white noise) is easily computed from (21) and is given by the formula

$$E[r_i^2] = \frac{\sigma_\xi^2}{\Delta t^4} + \sigma_f^2,$$

where we use the fact that $\tilde{L}_{3,i}$ is a column of an orthogonal matrix.

The processes $r_i$ and $\mathbf{q}_i$ are cross-correlated and their cross-covariance matrix is given by the expression:

$$E[r_i \mathbf{q}_i^{\mathrm{T}}] = \frac{1}{\Delta t^2}(0, 0, 0, \sigma_\xi^2).$$

Under the above assumptions, we now can formulate the optimal estimation problem (in the mean-square error sense) for the state vector $\mathbf{x}_i$ at each time instant given the state-space system (18) and measurements $y_i$, $i = 0, \ldots, n$. We assume that the estimate of the state vector at the initial time moment $\mathbf{x}_0$ is 0 and the initial covariance matrix $E[\mathbf{x}_0 \mathbf{x}_0^{\mathrm{T}}]$ is given. The optimal estimation algorithm is the Kalman filter and smoothing [13].

### *3.4. Theoretical Analysis of Gravity Estimation Accuracy*

Let us determine the gravity estimation accuracy when using the proposed approach (based on the refined model of kinematic acceleration error). For this, we derive an approximate expression for the transfer function of the optimal filter, which maps the measurement $y$ (9) to the gravity disturbance estimate (the so-called gravimetric filter).

First, we reduce equation (9) to stationary form by neglecting the systematic errors $k_E, k_N$ and accelerometer measurement noise $\mathbf{q}_f$. The gravity disturbance $\delta g$ and GNSS acceleration error $e_a$ are assumed here to be continuous-time stationary processes with given PSDs (10) and (14), respectively. Then the optimal (in the mean-square error sense) linear estimate of gravity is determined by a smoothing filter that is given in the frequency domain by the expression (the Wiener filter) [19]

$$W_1(\omega) = S_g(\omega)\,(S_g(\omega) + S_e(\omega))^{-1} = \left(1 + \left(\frac{\sigma_a}{\sigma_g}\right)^2 \omega^8\right)^{-1}. \tag{22}$$

In deriving (22), we used expression (14) in an approximate form as $\frac{\sigma_a^2}{2\pi}\omega^4$ for small $\omega$.

Thus, the gravimetric filter based on the refined GNSS acceleration error model (14) approximately corresponds to the two-pass Butterworth filter of order 4 (Fig. 2a). In Fig. 2a, also shown is the gravimetric filter $W_2(\omega)$ constructed using the simplified GNSS acceleration error model (white noise with intensity $\sigma_q^2$) and the same model for gravity:

$$W_2(\omega) = \left(1 + \left(\frac{\sigma_q}{\sigma_g}\right)^2 \omega^4\right)^{-1}. \tag{23}$$

The transfer function (23) corresponds to the two-pass Butterworth filter of order 2.

Let us determine the accuracy of gravity estimation using the constructed filters as the PSD of the estimate error, provided that the true PSD of gravity coincides with the a priori model (10)

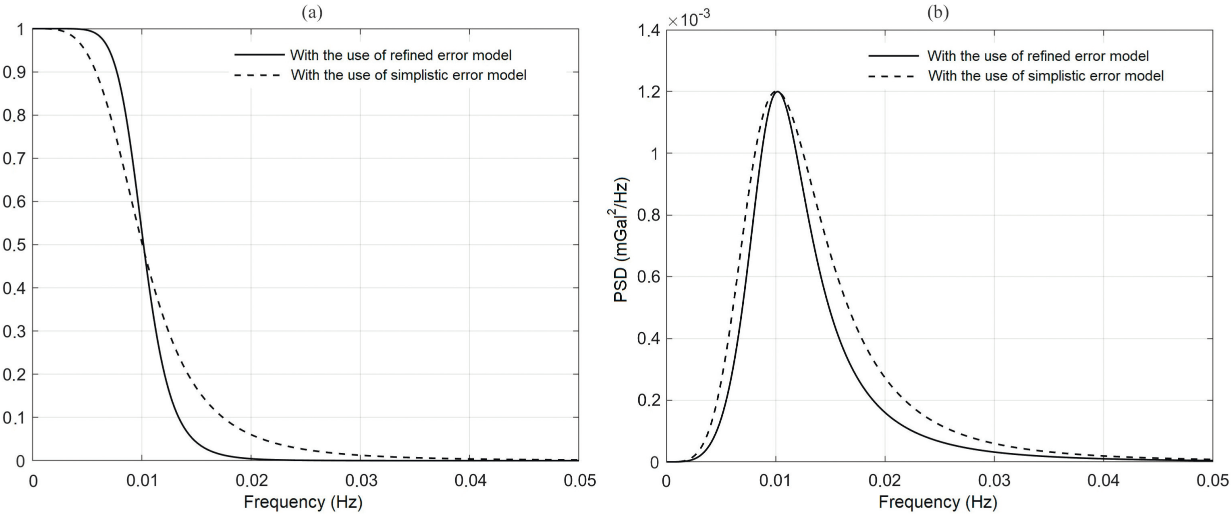


**Fig. 2.** (a) Transfer functions of the gravimetric filters in the new approach (based on the refined model of GNSS acceleration error) and in the common approach (based on a simplistic model of the GNSS acceleration error). (b) Power spectral densities of the errors in gravity estimates provided by the gravimetric filters, $\mathrm{mGal}^2/\mathrm{Hz}$ ($1\ \mathrm{mGal} = 10^{-5}\ \mathrm{m/s}^2$).

and the true PSD of the GNSS acceleration error coincides with the model (14). Then the PSD of the error of the gravity estimate provided by the gravimetric filter $W_1(\omega)$ is determined as

$$S_{\delta g}(\omega) = S_g(\omega)S_e(\omega)\left(S_g(\omega) + S_e(\omega)\right)^{-1} = \frac{\sigma_a^2\omega^4}{2\pi}\left(1 + \left(\frac{\sigma_a}{\sigma_g}\right)^2 \omega^8\right)^{-1}. \tag{24}$$

The PSD of the error of gravity estimate obtained using the gravimetric filter $W_2(\omega)$ is determined by:

$$\begin{aligned} S_{\delta g}(\omega) &= |1 - W_2(\omega)|^2 S_g(\omega) + |W_2(\omega)|^2 S_e(\omega) \\ &= \frac{\sigma_a^2\omega^4}{2\pi}\left(1 + \frac{\sigma_q^4}{\sigma_g^2\sigma_a^2}\right)\left(1 + \left(\frac{\sigma_q}{\sigma_g}\right)^2 \omega^4\right)^{-2}. \end{aligned} \tag{25}$$

The plots of PSDs (24)–(25) are shown in Fig. 2b. The figure shows that when using the filter based on the refined GNSS acceleration error model, the PSD of the gravity estimation error is smaller than when using the filter based on the simplified acceleration error model.

## 4. NUMERICAL RESULTS

To test the developed gravity estimation algorithm (Section 3.3), we used data from a state-of-the-art strapdown airborne gravimeter (iCORUS by iMAR) recorded on December 17, 2022 during a flight of an airborne gravity campaign. The gravimeter was flown along ten repeated lines (i.e., above the same ground track) during this flight (Fig. 3). The repeated lines are oriented in the east-west and west-east directions. The length of each line is about 110 km. The flight was carried out using a Cessna 208B aircraft at a constant altitude of 760 m above the reference ellipsoid. The average aircraft speed along survey lines was 70 m/s. The total flight duration is 7 hours. The gravimetry campaign was conducted by Aerogeophysica JSC (Russia) in the Krasnoyarsk Krai.

The gravimeter data included raw measurements from the IMU inertial sensors (at the data rate of 400 Hz) and raw measurements from the onboard and ground-based GNSS (GPS) receivers

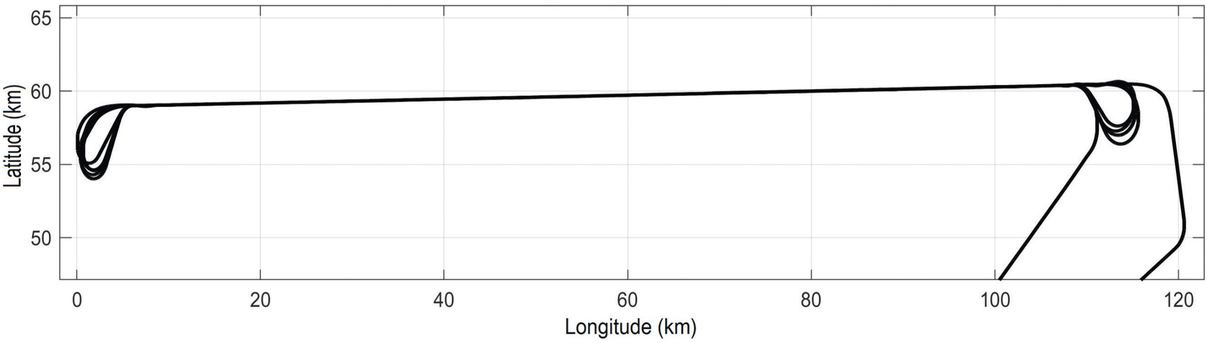


**Fig. 3.** Flight trajectory on the longitude-latitude plane (GNSS data).

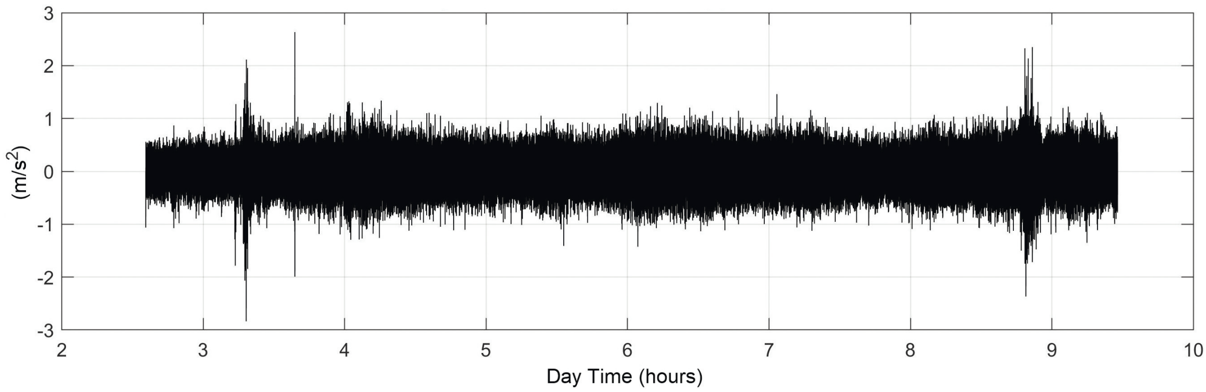


**Fig. 4.** Vertical kinematic accelerations from GNSS carrier-phase measurements, m/s$^2$.

from JAVAD (at 10 Hz). Preliminary stages of raw data postprocessing were performed using the software developed by the Faculty of Mechanics and Mathematics at Lomonosov Moscow State University [2] (software packages *INS-GNSS* and *IMU-GRAV* [20, 21]). Namely, the following tasks were solved at the initial stage:

1) computing the GNSS solutions (in the carrier phase differential mode), which included calculation of

- position (latitude, longitude and height above the reference ellipsoid) of the antenna of the onboard GNSS receiver;
- velocity (east, north and vertical components) of the onboard receiver;
- kinematic accelerations (east, north and vertical components) of the onboard receiver;

2) computing integrated IMU/GNSS solutions, which included estimation of

- IMU attitude angles (heading, roll and pitch);
- systematic errors of the IMU inertial sensors.

The vertical kinematic accelerations calculated from the carrier phase measurements using the algorithm from [10] are shown in Fig. 4.

The gravity estimate along the flight trajectory was computed using the proposed algorithm based on the refined GNSS acceleration error model. The gravity estimation accuracy was determined based on the statistics from ten repeated flight lines. The root-mean-square (RMS) value is 0.706 mGal. The gravity estimates at the repeated lines are shown in Fig. 5.

For comparison, another estimate of the gravity disturbance was obtained using the standard approach, which is based on the simplified GNSS acceleration error model (white noise). The same stochastic models were used for the residual attitude errors $k_E$, $k_N$, accelerometer measurement

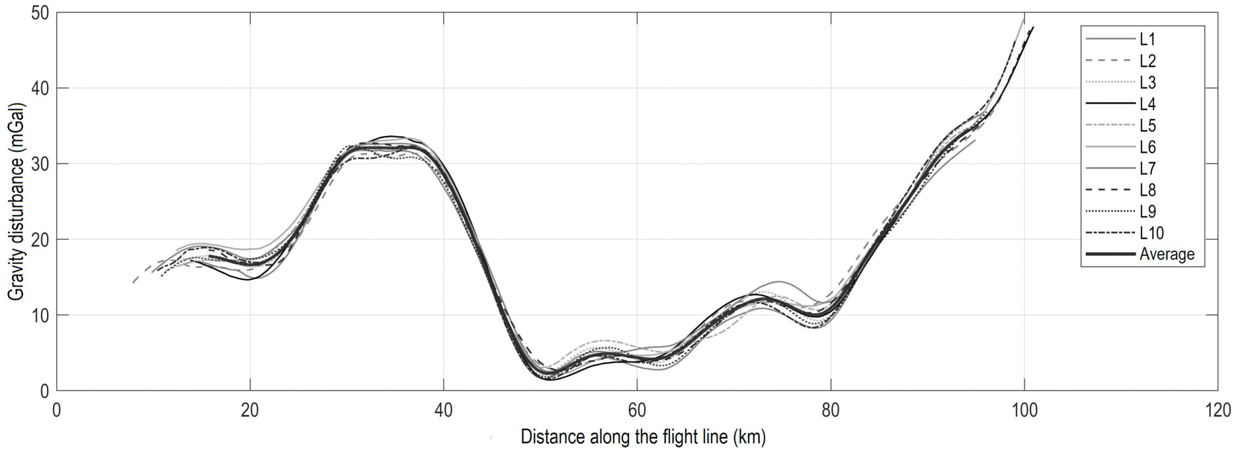


**Fig. 5.** Gravity disturbance estimates at the repeated lines provided by the new algorithm based on the refined model of GNSS acceleration error, mGal.

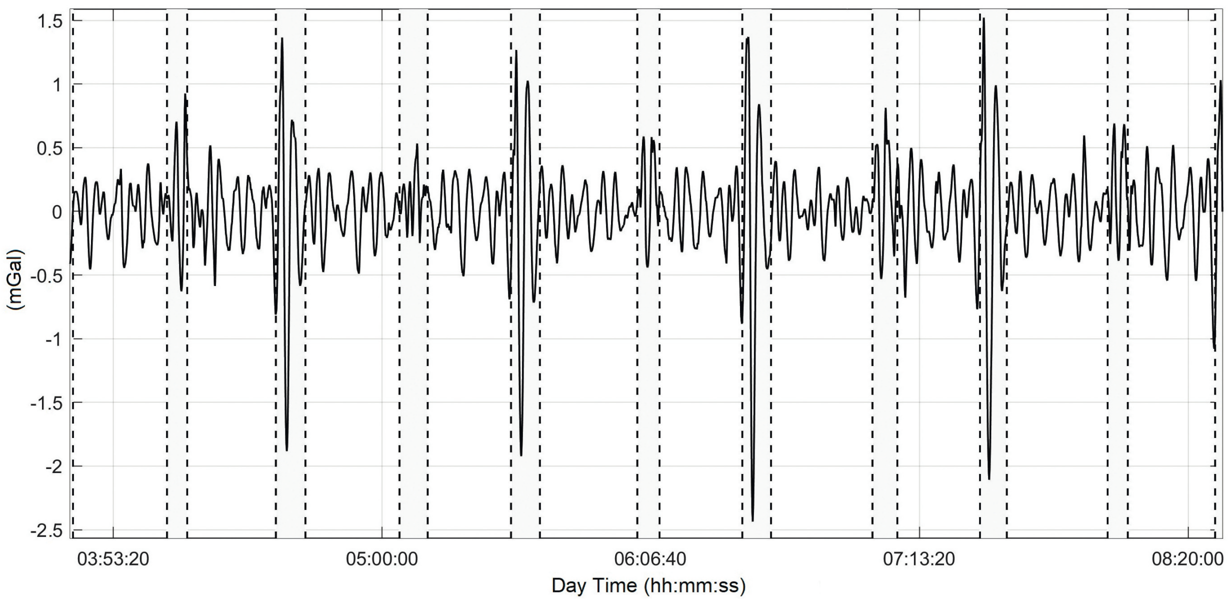


**Fig. 6.** Difference between the gravity disturbance estimates obtained in the new and standard approaches, mGal.

noise $\mathbf{q}_f$ and gravity disturbance in (9) as in the proposed approach. The estimation algorithm in the standard approach is the Kalman filter and smoothing. As shown above, the transfer function of the gravimetric filter in the standard approach is close to the 2nd-order two-pass Butterworth filter (Fig. 2a). The accuracy of the gravity estimates obtained in the standard approach was also determined based on the statistics from ten repeated lines and is 0.749 mGal (RMS). This demonstrates a slightly worse repeatability of the gravity estimates compared to the results from the proposed approach (based on the refined GNSS acceleration error model).

In Fig. 6, the difference between the gravity estimates computed by the proposed and standard approaches is shown. The standard deviation of the difference is 0.764 mGal, which is quite significant. We attribute this discrepancy to the gravity estimation error introduced by the standard approach as it showed worse repeatability of the gravity estimates at the repeated lines.

The absolute values of the difference between the gravity estimates provided by the two approaches reach 2.5 mGal, with maxima occurring during the aircraft turns (each lasting from 5 to 8 minutes) between the repeated lines (visible as spikes in Fig. 6). This can probably be explained by the fact that during aircraft turns, GNSS acceleration errors have a wider frequency range and are more effectively suppressed by the gravimetric filter of the proposed approach, which has a steeper roll-off near the cutoff frequency (Fig. 2a).

## 5. CONCLUSIONS

The gravity estimation algorithm based on the refined model of errors in the kinematic accelerations computed from the GNSS carrier phase measurements has been proposed. The refined error model takes into account the specificity of the method that was used for computing the kinematic accelerations and is defined in the time domain as the second central difference of a discrete-time white noise process.

The proposed approach was compared with the standard one, which uses a simplified GNSS acceleration error model (white noise). The approaches were compared based on processing raw data from a strapdown gravimetry flight (ten repeated lines). The results show a higher accuracy of the gravity estimates provided by the proposed approach, which is 0.71 mGal (RMS), while the standard approach showed the 0.75 mGal accuracy. The difference between the gravity estimates obtained by the two approaches reaches 2.5 mGal and is attributed to the estimation errors introduced by the standard approach.

Based on the obtained test results, we conclude that the proposed approach seems promising for implementing in postprocessing software packages of state-of-the-art strapdown airborne gravimeters.

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